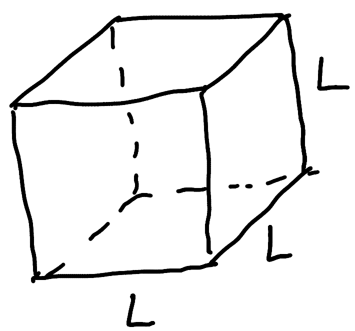
Computing the number of states and the DoS

Consider a cube with periodic boundary conditions (the boundary conditions do not really matter)

$\hbar = 1$



$$\psi = \frac{1}{L^{\frac{3}{2}}} e^{i(k_x x + k_y y + k_z z)}$$

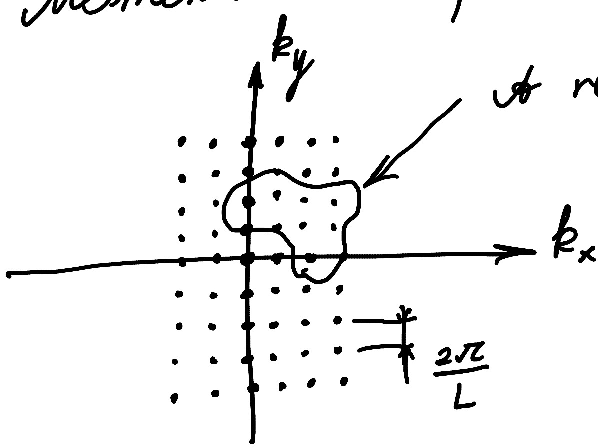
Boundary conditions:

$$\begin{cases} k_x L = 2\pi n_x \\ k_y L = 2\pi n_y \\ k_z L = 2\pi n_z \end{cases}$$

$$n_i \in \mathbb{Z}$$

Note that n may be both negative and positive

→ Momenta are quantised: $\Delta k_i = \frac{2\pi}{L}$



A region of momentum space $d\mathbf{k}$

\# momentum states

$$= V \frac{d\mathbf{k}}{(2\pi)^3}$$

If we want to count all the states inside the Fermi surface

concentr.

$$V \int \frac{d\mathbf{k}}{(2\pi)^3} = N \rightarrow n = \int \frac{d\mathbf{k}}{(2\pi)^3}$$

$$V \int_{FS} \frac{d\mathbf{k}}{(2\pi)^3} = N \rightarrow n = \int \frac{d\mathbf{k}}{(2\pi)^3}$$

The density of states

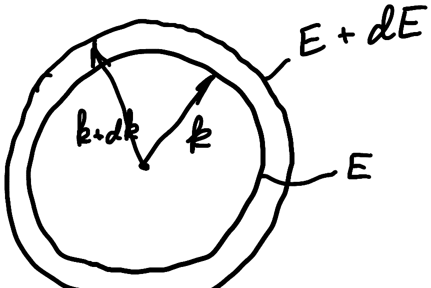
Usually defined as $\rho(E) = \frac{dN(E)}{dE}$

! but! in condensed matter defined per unit volume:

$$\nu(E) = \frac{dn(E)}{dE}$$

The DoS for a system with a spherically symmetric dispersion

$$n(E) = \int \frac{d\mathbf{k}}{(2\pi)^3} \, \theta(E - \varepsilon_{\mathbf{k}})$$ - concentration of quasiparticles with energies $\varepsilon_{\mathbf{k}} < E$



$$\frac{4\pi k^2 dk}{(2\pi)^3} = \nu \cdot dE$$

$$v = \frac{dE}{dk}$$

$$\nu = \frac{4\pi k^2}{(2\pi)^3 v} = \frac{k^2}{2\pi^2 v}$$

If there is spin, this needs to be multiplied by 2

$$\rightarrow \nu = \frac{k^2}{\pi^2 v}$$

$$\rightarrow \quad \nu = \frac{k^2}{\pi^2 v}$$